Shape Estimation Using Sun-angle Measurements Obtained from Distributed Sensors on the Surface of a Solar Sail

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This paper proposes a novel approach to monitor the shape of a solar sail using Sun-angle measurements obtained directly on the sail membrane. These measurements can be easily provided by already installed thin-film solar arrays or dedicated, low-impact sensors. A general non-linear least-squares formulation is first presented, and then reduced to a linear least-squares problem for large sails. A rank deficiency is observed in the estimation method, but is avoided by considering measurements at multiple epochs, reconstructing a time-averaged shape. Multi-particle models (MPM) of JAXA’s IKAROS and Solar Power Sail (SPS) are used to evaluate the performance of the method. The SPS shape can be accurately estimated using this method. The deviation from the MPM is within 0.1 m, except near the corners where it increases to 0.2 m. Including additional sensors in these locations further improves accuracy. Shape estimation was also performed on a moderately-wrinkled IKAROS model to evaluate the algorithm for higher distortions. A maximum error of 0.5 m in the inner perimeter corners of the sail and 0.3 m near the wrinkles is observed, partially due to low resolution of the MPM. The presented results suggest that the proposed method can easily provide continuous shape estimation for future solar sail missions.

Key Words: Solar Sail, Shape Estimation, Shape Monitoring, Distributed Sensors, Sun-angle

Nomenclature

Symbols

- $x, y$: solar sail in-plane coordinates
- $z$: solar sail out-of-plane deformation
- $n$: local normal unit-vector to a point of the solar sail
- $s$: unit Sun vector
- $\beta$: local Sun-angle at a point of the solar sail
- $\phi$: in-plane Sun-angle at a point of the solar sail
- $r$: coordinates of a point in the solar sail
- $\xi, \eta$: Monge patch parameters
- $a$: solar sail shape series coefficient vector
- $h$: vector space basis evaluated at point of the surface
- $A$: observation matrix for least-squares method
- $b$: measurement vector for least-squares method
- $n$: number of sensors
- $m$: number of epochs with available measurements
- $U$: photovoltaic cell voltage
- $T$: temperature
- $\sigma$: Stefan-Boltzmann constant
- $\dot{q}$: solar power flux density
- $\alpha$: (solar) absorptance
- $\varepsilon$: (IR) emittance
- $C_h$: sensor heat capacity
- $\lambda$: thermal conductance
- $d$: thermal sensor thickness
- $S_L$: form factor

Superscripts

- $*$: measured quantity
- $j$: value at epoch $j$

Subscripts

- $x, y, z$: vector component
- $\xi, \eta$: derivative
- $b$: spacecraft body fixed frame
- $i$: value at sensor $i$

1. Introduction

The ability to continuously monitor the membrane shape of a solar sail is paramount to provide reliable and predictable orbit and attitude control. This becomes more significant, as missions with increasing complexity and extended duration consider the use of solar or solar-electric sails. For solar sailing, knowledge of the direction and magnitude of the accelerating force resulting from solar radiation pressure (SRP) is critical. Solar-electric sails provide a large surface area to generate electric power from thin-film solar cells. They can drive high specific impulse ion-engines, even in the outer solar system. 1) Monitoring and predicting the shape of these spacecraft is important to keep track of the power system performance.

Sail shape can be estimated a posteriori from flight data. 2) However, to perform orbit and attitude control, real-time shape information must be available. So far, solar sail spacecraft have mainly relied on images from optical cameras (Fig. 1).

(a) IKAROS body-mounted and detachable cameras. [JAXA]
(b) Lightsail deployable cameras. [The Planetary Society]

Fig. 1. IKAROS (a) and Lightsail-1 (b) mounted camera systems.

These systems include cameras mounted on the spacecraft main body or on deployable structures, as well as fully de-
tachable systems which are released from the main spacecraft. \(^4,5,6\) Images captured by these cameras have been used to perform shape estimation of solar sails. \(^7,8,9\)

However, both mounted and detachable systems have drawbacks. Mounted systems either have a restricted view of the sail membrane (Fig. 2a), or require extending camera mounts to achieve a good view point (Fig. 2b). These extending systems become impractical when applied to large solar sails, as the spacecraft main body (especially height) does not necessarily scale with sail size. One example of such a large sail is JAXA’s next generation Solar Power Sail (SPS) mission. \(^1\) Its sail features a surface area of around 2500 m\(^2\); more than 10 times that of IKAROS (200 m\(^2\)).

![Image](image1.png)

(a) [JAXA]  (b) [The Planetary Society \(^3\)]

Fig. 2. IKAROS (a) and Lightsail-1 (b) mounted camera images.

Systems such as the Deployable-Camera (DCAM) used on the IKAROS mission can capture sails with large surface areas (Fig. 3). However, they only provide a one-shot option, and cannot continuously monitor sail shape to detect changes over time. Shape estimation from these systems also requires accurate reconstruction of the camera trajectory. \(^10\)

![Image](image2.png)

(a) DCAM and its separation mechanism. [JAXA]  (b) IKAROS sail captured by DCAM. [JAXA]

Fig. 3. IKAROS DCAM (a) and sail image (b).

This paper proposes a novel method to monitor the global shape of a solar sail from measurements of the Sun-angle \(\beta\), taken directly on the sail membrane. Measurements can be obtained by already available solar arrays, or from a grid of photo-voltaic or thermal sensors, distributed on the sail surface. Here, we will focus on the application of this method to square-shaped, spinning-type solar sails such as IKAROS (Fig. 4) and SPS (Fig. 5).

2. Sun-angle based shape estimation

Let \((x, y, z)\) be a body-fixed, orthogonal reference system centred at the square solar sail whose shape we want to estimate. We define the \(z\) direction perpendicular to the undeformed sail surface, and the \(x\) and \(y\) directions parallel to the sail sides. Assuming that the in-plane deformation of the sail membrane is negligible, we seek to estimate the out-of plane deformation. The sail half-side is used as unit of length.

The direction of the Sun vector \(s = (s_x, s_y, s_z)\) is defined by two angles on the main body (Fig. 6): the in-plane angle \(\phi_n\), measured from the \(x\) axis; and the global Sun-angle \(\beta_n\), which corresponds to the angle between \(s\) and the normal of the \(x-y\) plane \(n_t\). The (local) Sun-angle \(\beta\) is the angle between \(s\) and the local surface normal \(n\). It can be measured directly on the sail surface with little effort (Section 3.).

![Image](image3.png)

(a) Sep Camera 1 (DCAM)  (b) IKAROS sail captured by DCAM. [JAXA]

Fig. 3. IKAROS DCAM (a) and sail image (b).

The sail surface can be characterized by the Monge patch

\[
\begin{align*}
\mathbf{r}(\xi, \eta) &= \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} \xi \\ \eta \\ z(\xi, \eta) \end{pmatrix}, \\
&\quad (\xi, \eta) \in [-1, 1] \times [-1, 1]. \quad (1)
\end{align*}
\]

This parametrization is regular because the partial derivatives with respect to the parameters \((\xi, \eta)\) are linearly independent:

\[
\mathbf{r}_\xi = \frac{\partial \mathbf{r}}{\partial \xi} = \begin{pmatrix} 1 \\ 0 \\ \frac{\partial z}{\partial \xi} \end{pmatrix}, \quad \mathbf{r}_\eta = \frac{\partial \mathbf{r}}{\partial \eta} = \begin{pmatrix} 0 \\ 1 \\ \frac{\partial z}{\partial \eta} \end{pmatrix}. \quad (2)
\]
The normal vector to the sail surface in an arbitrary point is
\[ n = \frac{\mathbf{r}_\xi \times \mathbf{r}_\eta}{\|\mathbf{r}_\xi \times \mathbf{r}_\eta\|} = \left(\frac{\partial z}{\partial \eta}, -\frac{\partial z}{\partial \xi}, 1\right)^T / \sqrt{1 + \left(\frac{\partial z}{\partial \eta}\right)^2 + \left(\frac{\partial z}{\partial \xi}\right)^2}. \]

The infinite power series expansion of the sail z-component about the sail centre \((0,0)\) reads
\[ z(\xi, \eta) = \sum_{k=0}^{\infty} \sum_{t=0}^{k} \frac{1}{k!} \left(\frac{\partial z}{\partial \eta}\right)^k \xi^k \eta^{t-k}. \]

We can truncate the series to the \(k_{\text{max}}\) power, reorder the terms in the summation, and redefine the coefficients to yield
\[ z(\xi, \eta) \approx \sum_{k=0}^{k_{\text{max}}} h_k(\xi, \eta) a_k = h^T (\xi, \eta) a, \]  \hspace{1cm} (5)

where \(h^T a\) is the inner product of the two vectors, and
\[ h(\xi, \eta) = \left(1, \xi, \eta^2, 2\xi\eta, \eta^3, 3\xi^2\eta, 3\xi\eta^2, \ldots\right)^T. \]  \hspace{1cm} (6)

The partial derivatives of \(z\) take the simple form
\[ \frac{\partial z}{\partial \xi} = h_1^T a, \quad \frac{\partial z}{\partial \eta} = h_2^T a, \]  \hspace{1cm} (7)

where
\[ h_\xi = \frac{\partial h}{\partial \xi} = \left(0, 1, 0, 2\xi, 2\xi, 0, 3\xi^2, 6\xi\eta, 3\eta^2, \ldots\right)^T, \]  \hspace{1cm} (8)
\[ h_\eta = \frac{\partial h}{\partial \eta} = \left(0, 0, 1, 0, 2\eta, 2\eta, 0, 3\xi^2, 6\xi\eta, \ldots\right)^T. \]  \hspace{1cm} (9)

The Sun-angle \(\beta\) at a point \((x, y) = (x(\xi), y(\eta))\) on the sail surface can be determined as the angle between the Sun vector \(s\) and the local normal \(n\):

\[ \cos \beta = \mathbf{n} \cdot \mathbf{s} = \frac{s_z - s_x h_\xi^T a - s_y h_\eta^T a}{\sqrt{1 + (h_\xi^T a)^2 + (h_\eta^T a)^2}}. \]  \hspace{1cm} (10)

The gradient of \(\cos \beta\) reads
\[ \frac{\partial \cos \beta}{\partial \mathbf{a}} = -\frac{(s_x + h_\xi^T a)(h_\xi)^2 + (s_y + h_\eta^T a)(h_\eta)^2}{(1 + (h_\xi^T a)^2 + (h_\eta^T a)^2)^{3/2}}. \]  \hspace{1cm} (11)

By evaluating Eq. (12), we obtain the simple expression
\[ \cos \beta (a) \approx s_z - s_x h_\xi^T a - s_y h_\eta^T a. \]  \hspace{1cm} (13)

Eq. (11) can then be reduced to:
\[ \minimize \| \mathbf{Aa} - \mathbf{b} \|^2 \]  \hspace{1cm} (14)

where
\[ \mathbf{A} = \begin{bmatrix} s_x h_\xi^T (\xi_1, \eta_1) + s_y h_\eta^T (\xi_1, \eta_1) \\ \vdots \\ s_x h_\xi^T (\xi_n, \eta_n) + s_y h_\eta^T (\xi_n, \eta_n) \end{bmatrix}, \quad \mathbf{b} = \begin{bmatrix} s_z - \cos \beta_1^* \\ \vdots \\ s_z - \cos \beta_m^* \end{bmatrix}. \]  \hspace{1cm} (15)

The linear least-squares problem has the analytical solution
\[ \mathbf{a} = (\mathbf{A}^T \mathbf{A})^{-1} \mathbf{A}^T \mathbf{b}. \]  \hspace{1cm} (16)

\(\dagger\) represents the Moore-Penrose pseudo-inverse operation, which gives among all the possible solutions, the one with the minimum value of the objective function. (12)

Once \(\mathbf{a}\) is determined, the sail out-of-plane deformation can be calculated at any given point \((x, y)\) using Eq. (5).

However, inspection of both the linear \((\mathbf{A}^T \mathbf{A})\) matrix and the non-linear formulations in Eq. (11) reveals a rank deficiency problem, in case only Sun-angle measurements from a single epoch are used. That is, the shape of the sail membrane cannot be accurately estimated from Sun-angle measurements taken only at a single point in time.

For rotating solar sails, this problem can be solved by considering two or more sets of Sun-angle measurements, taken at different times. The \(h_\xi\) and \(h_\eta\) vectors evaluated at each sensor location do not evolve with time, but the Sun direction and the Sun-angle measurements are different at each epoch. Considering Sun vectors \(s^j = (s^j_x, s^j_y, s^j_z)^T\) and Sun-angle measurements \(\cos \beta^*_j\) at the epochs \(j = 1, \ldots, m\), the non-linear formulation becomes:

\[ \minimize \sum_{i=1}^n \sum_{j=1}^m \left(\cos \beta_i^j (a) - \cos \beta^*_j\right)^2 \]  \hspace{1cm} (17)

For the linear least-squares estimation, \(\mathbf{A}\) and \(\mathbf{b}\) are constructed by vertically concatenating their single-epoch equivalents:

\[ \mathbf{A} = \begin{bmatrix} s^1_x h_\xi^T (\xi_1, \eta_1) + s^1_y h_\eta^T (\xi_1, \eta_1) \\ \vdots \\ s^m_x h_\xi^T (\xi_n, \eta_n) + s^m_y h_\eta^T (\xi_n, \eta_n) \end{bmatrix}, \quad \mathbf{b} = \begin{bmatrix} s_z - \cos \beta_1^* \\ \vdots \\ s_z - \cos \beta_m^* \end{bmatrix}. \]  \hspace{1cm} (18)
3. Sensors

3.1. Photovoltaic sensors

One type of sensor to obtain Sun-angle measurements is likely to already be installed on the membrane of some solar sail spacecraft in the form of thin-film solar cells. Readings from Sun sensors and solar arrays are commonly used to determine orientation of a satellite towards the Sun. The output of a solar cell is the result of local solar power flux density \( \dot{q} \), Sun-angle \( \beta \), and cell efficiency. Using Lambert’s cosine law, \( \beta \) can be calculated from the measured voltage \( U \) and the calibrated Voltage \( U_0 \) at zero-degree Sun-angle:

\[
U = U_0 \cos \beta = U_0 \mathbf{s} \cdot \mathbf{n} \tag{20}
\]

If the existing solar cells are too few to provide sufficient measurements, small sized sensors can be placed at additional locations on the sail. In case thin-film solar cells are used as sensors, they have little impact in terms of size and weight. Photovoltaic cells (in rigid form) have been implemented as dedicated attitude determination sensors in missions such as the MASCOT asteroid lander (Fig. 7).

![Fig. 7. Photoelectric Cell Sensor (PEC) used for the attitude determination of the MASCOT asteroid lander. [DLR 14] ]

Depending on the size and flexibility of the panel/sensor, Sun-angle measurements can be interpreted as single point measurements, or as averaged values over the sensor area. Sensor accuracy depends on the initial calibration with \( U_0 \). Temperature dependent cell efficiency can also affect accuracy. For long-term missions, degradation effects may have to be considered.

3.2. Temperature sensors

The Sun-angle on a sail can also be measured using temperature sensors, so called orientation temperature sensors (OTS). When the optical properties of a surface (\( \varepsilon, \alpha \)) are well defined, sensor temperature \( T_{sensor} \) relates directly to \( \beta \), as Lambert’s cosine law for solar power flux density applies here as well.

The thermal differential equation for a measurement taken at the centre of a square shaped sensor (Fig. 8) is

\[
C_h \frac{\partial T_{sensor}}{\partial t} = A \dot{q} \alpha \cos \beta + \lambda d S_L (T_{tail} - T_{sensor}) + A \sigma (\varepsilon_{front} + \varepsilon_{back}) (T_{space}^4 - T_{sensor}^4) \tag{21}
\]

Eq. (21) accounts for illumination of the sensor surface with Sun-angle \( \beta \) and solar power flux density \( \dot{q} \), radiative heat exchange with Space from both sides of the sail, and thermal conduction between the sensor and sail membrane using a form factor \( S_L \). The heat capacity \( C_h \) determines how fast the sensor reacts to changes in the Sun-angle. Table 1 lists the sensor parameters used in this study, based on existing data.

![Fig. 8. Temperature sensor schematic for possible top or bottom attachment; dimensions and concept adapted from OTS design. 16]

<table>
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<th>parameter</th>
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<th>value</th>
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<tr>
<td>( \alpha )</td>
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<td>0.90</td>
</tr>
<tr>
<td>( \varepsilon_{front} )</td>
<td>–</td>
<td>0.82</td>
</tr>
<tr>
<td>( \varepsilon_{back} )</td>
<td>–</td>
<td>0.035</td>
</tr>
<tr>
<td>( C_h )</td>
<td>J K(^{-1})</td>
<td>0.024</td>
</tr>
<tr>
<td>( \lambda )</td>
<td>W m(^{-1}) K(^{-1})</td>
<td>( 1.2 \times 10^{-5} )</td>
</tr>
<tr>
<td>( d )</td>
<td>( \mu m )</td>
<td>120</td>
</tr>
<tr>
<td>( A )</td>
<td>m(^2)</td>
<td>( 1.0 \times 10^{-4} )</td>
</tr>
<tr>
<td>( S_L )</td>
<td>–</td>
<td>4.12</td>
</tr>
</tbody>
</table>

The conductive term in Eq. (21) is considered to be an unknown thermal input, as sail temperature is generally not known. However, heat transfer between the sail and sensor remains small, independent of sail temperature \( T_{tail} \). The reason for this is the small thickness \( d \) of the sensor, in relation to its area \( A \). Consequently, the conductive term can be neglected, resulting in a maximum error of only \( \pm 0.15 \) deg in \( \beta \).

For steady-state measurements, the capacitive term is considered to be zero. This simplification introduces some error due to delay in the temperature progression of the sensor. However, this term would only be of use if the entire temperature history of the sensor was available. The Sun-angle can then be obtained by solving:

\[
\cos \beta = \frac{\sigma (\varepsilon_{front} + \varepsilon_{back})}{\alpha \dot{q}} (T_{space}^4 - T_{sensor}^4) \tag{22}
\]

For the sensor surface, any material with known properties (\( \alpha, \varepsilon \)) is applicable, including the original membrane. However, selecting one that has high \( \alpha \) and \( \varepsilon \) (black body) leads to faster steady-state temperatures. The maximises the heat exchange rate between the sensor and its environment.

4. Solar sail and sensor models

4.1. Sail shape and sensor location/distribution

To investigate the performance of the newly developed method and proposed sensors, we use a number of solar sail models based on JAXA’s IKAROS and SPS missions. The sail geometry is obtained from ongoing simulations, based on established multi-particle models (MPM). Fig. 9 shows the steady-state shape of the SPS model used in the shape estimation.

The SPS model considers membrane distortion due to centrifugal forces from its rotation rate (0.055 rpm), and the effect
of SRP. Placement and properties of the thin-film solar cells are based on the current development state.\(^{10}\) Due to the large surface area used for solar power generation, the installed solar cells offer sufficient measurement points. Locations with no solar cells are the petal centre-line and the outer perimeter folds. These are reserved for the launch-lock mechanism and the steering devices.

In case of IKAROS, the number of attached thin-film solar cells does not provide a sufficient number of Sun-angle measurements or coverage (Fig. 4). We therefore place a grid of distributed sensors on the sail membrane for testing the shape estimation method (Fig. 10). These sensors can either be solar cell or thermal sensors.

4.2. Thermal simulation of sail and sensors

While the solar cells are considered to provide direct Sun-angle measurements, the temperature sensor readings first need to be converted to Sun-angles $\beta$.

To reproduce sensor behaviour under actual missions conditions a full thermal simulation based on Eq. (21) is performed for each sensor. The required sail temperature $T_{\text{sail}}$ near the sensors is obtained from separate thermal simulations of the rotating solar sails (Fig. 11). Sun-angles are then calculated from the simulated sensor temperatures $T_{\text{sensor}}$ via Eq. (22).

5. Application of the shape estimation method

Using the set of sensors defined in the previous section, the sail shape for the SPS and IKAROS missions is estimated.

5.1. Solar Power Sail

In case of the SPS, the large sail dimensions in $x, y$ make it possible to apply the linear least-squares estimation method. No difference is hereby observed between the results of the linear and non-linear method. Fig. 12 shows the estimated sail shape considering measurements over a full rotation of the spacecraft. While the error between the MPM data and estimation remains below 0.1 m in most locations, large errors of 0.2 m are found near the tip masses and bridges. This can be explained by the lack of sensors in these regions.

5.2. IKAROS

Compared to the SPS, IKAROS has much smaller dimensions. To test the performance of the method, MPM data with a much larger out-of-plane deformation is used. For this case the non-linear estimation yields improved results over the linear method. Fig. 14 shows the reconstructed IKAROS sail shape using thermal sensors.
While the overall result is satisfactory, some moderately-high local errors are introduced due to the highly distorted nature of the MPM data, namely in the corners of the inner perimeter and at the wrinkled parts of the membrane (see Fig. 10). Use of higher resolution MPM data should be employed in the future, to determine the error of the proposed method for a moderately wrinkled sail more accurately.

To compare sensor performance, the thermal sensor shape estimation of IKAROS is compared to its zero-delay counterpart. The differences are shown in Fig. 15. Regions where the deformation was overestimated correspond to high z coordinate points, shifted in location by a rotation in opposite direction to the spacecraft’s angular velocity. This is likely caused by the measurement delay of the thermal sensors due to their non-zero heat capacity. The 30 s delay of the sensor corresponds to a 10 deg rotation of the sail at 0.55 rpm. In this particular configuration the maximum error introduced by these sensors is bounded by 0.15 m. Further performance improvements are possible by adapting the OTS design to the specific use on solar sails.

6. Conclusions

The results of the sail shape estimation performed on SPS and IKAROS show that the proposed method is able to correctly, and continuously estimate the shape of a solar sail using only Sun-angle measurements. Thus, adopting this new method in future solar sail missions can provide accurate, continuous shape-monitoring capabilities with minimum impact on the solar sail design.

The observed problem of rank deficiency is solved by taking measurements at different times during the rotating motion of the spinning-type solar sail spacecraft. For the SPS, the existing solar arrays can provide sufficient measurement points to perform shape estimation. The large size of the sail with respect to its deviation in the out-of-plane direction results in very accurate estimates (maximum error of 0.2 m near the tip masses and bridges, 0.1 m everywhere else). The addition of sensors in few selected locations can further improve accuracy. In case of the moderately wrinkled IKAROS model, the applied distributed sensor grid is also capable of estimating the sail shape. The performance of the thermal sensors compared to photo electric sensors shows a maximum error of 0.15 m on the highly deformed IKAROS sail, caused by a 30 second delay due to sensor heat capacity.

In future works, there are several points that should be studied in higher detail. In the first place, more detailed trades between the photovoltaic and thermal sensor types should be performed. Thermal sensors offer a low-cost option with high reliability, especially if response time is reduced by adapting the OTS design for application on solar sail membranes. Next, applying this method to non-rotating solar sails is a major milestone for future work; possibly requiring the combination of different sensor types to deal with the observed rank deficiency. Finally, since the surface out-of-plane deformation was modelled with a simple Taylor expansion, the use of more sophisticated methods to represent the surface (i.e. splines or radial basis functions) should be investigated.

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