JAXA’s Trojan Asteroids Mission: Trajectory Design of the Solar Power Sail and its Lander

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In this paper we use dynamical system tools to design trajectories in the vicinity of Trojan asteroids for both: JAXA’s solar power sail and its lander. The current JAXA baseline considers a solar power sail hovering a Trojan asteroid at 40 km from its surface. Which will then descend to 1 km of the surface to release the lander. First we will exploit solar radiation pressure to place a solar power sail in orbit around the asteroid and illustrate how the effects of changing the sail orientation can enhance the hovering opportunities. Second we will focus on the lander release, performing a sensitivity analysis on its deployment velocity together with possible bouncing trajectory. To model the dynamics of the solar power sail and the lander we use the augmented Hill three body problem for the far gravity field dynamics and a perturbed two-body problem approximating the asteroid’s as triaxial ellipsoid for the close gravity field dynamics.

Key Words: Solar sail, Far vs near field dynamics, Augmented Hill problem, Triaxial ellipsoid, Lander.

1. Introduction

Jovian Trojans are asteroid located in the $L_4/L_5$ equilibrium points of the Sun-Jupiter system. Little is known about their origin and their evolution as the observations provide limited clues about the source of these asteroids. Currently, these asteroids are completely unexplored and the future JAXA’s sample and return mission to the Trojan asteroids will revolutionise our understanding of these bodies. Trojan asteroids are believed to either, being captured during the migration mechanism of Giant Planets, or formed during the planetary formation\textsuperscript{19} Their orbits are also destabilised by collision ejection. Thus, Trojans could also be the remnants of a much more substantial initial population of trapped bodies, or these objects could continually be replenished from an unidentified external source.\textsuperscript{3)} A mission to the Trojan asteroids represents the new frontier in mission exploration, and their study would also explain what happened to their former pristine formation.\textsuperscript{13} With respect to NEOs exploration, a space mission to Trojan asteroids faces the challenge of reaching the unexplored far side of our solar system where large amount of fuel is required and an efficient power supply is difficult.\textsuperscript{14)} JAXA’s Trojans assessment study concluded that the solar power sail is the best way to perform such a challenging mission.

This paper covers two different aspects of a solar sail mission to the Trojan asteroids, analysing possible trajectories to hover the asteroid and the landing of a small probe on the surface of the asteroid. Hence, we address both: far and near operations at the asteroid, dealing with different dynamical models that include the relevant perturbations in each scenario.

The current JAXA’s baseline proposes hovering the asteroid with an Earth-pointing solar sail.\textsuperscript{14)} However, exploring the effect of Solar Radiation Pressure (SRP) can enhance interesting orbit solution for a solar sail. For this purpose we study the dynamics of the sail in the far gravity field, where the asteroid is approximated as a point mass and the effects of SRP and Sun’s gravity are also taken into account, considering the augmented Hill 3 body problem as a model. In previous works\textsuperscript{2,7,8)} terminator orbits and other quasi-periodic orbits that appear in the system are proposed for hovering and monitoring an asteroid. In Section 3, we will review some of these orbits, moreover, by changing the sail orientation (keeping it fixed with respect to the Sun-sail line) we can artificially displace these orbits changing the visibility conditions, enlarging the monitoring sites.

The solar power sail will be equipped with a lander that will be released at 1 km from the surface of the asteroid. In this case, the dynamics of the lander is mainly influenced by the irregular shape of the asteroid.\textsuperscript{1} Hence, for the near field dynamics we consider the perturbed two-body problem where we include spherical harmonics for the irregular shape of the asteroid and the SRP effect is neglected. In Section 4, we discuss how to select the initial velocity at which the lander is released to guarantee a bounded motion of the lander around the asteroid.\textsuperscript{16)} This will prevent bouncing trajectories of the lander after touchdown, as in the Rosetta mission, with the potential risk of escaping from the asteroid surface.

2. Asteroid Parameters

Little is known on physical properties of Trojans asteroids due to the challenges one faces when observing and taking measurements of these objects from Earth. In this paper, we consider as an example the Trojan asteroid 2001 DY103 to set the orbital period and mean Sun distance (although most asteroids in the Trojan region present similar values). However, to determine the spin rate, shape and density we have used some statistics on the Trojan population.\textsuperscript{14)} These physical parameters are summarised in Table 1. Moreover, we approximate the shape of the asteroid by a triaxial ellipsoid where $a$, $b$ and $c$ (satisfying $a > b > c$) are the semi-major axis of the ellipsoid along the $x$, $y$ and $z$ axis respectively using an Asteroid-Centred Asteroid-Fixed (ACAF) reference frame. Table 2 contains the corresponding Stockes coefficients for this triaxial ellipsoid.
Table 1: Estimated physical parameters for asteroid 2001 DY103.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
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</thead>
<tbody>
<tr>
<td>Orbital period ($T$)</td>
<td>4343.87 days</td>
</tr>
<tr>
<td>Mean Sun-Asteroid Distance ($R$)</td>
<td>5.21 AU</td>
</tr>
<tr>
<td>Rotational Period ($\tau$)</td>
<td>10 hours</td>
</tr>
<tr>
<td>Effective Shape</td>
<td>$a = 11$, $b = 10$, $c = 9$ km</td>
</tr>
<tr>
<td>Effective Radius ($r_{ab}$)</td>
<td>9.97 km</td>
</tr>
<tr>
<td>Density ($\rho$)</td>
<td>2000 kg/m$^3$</td>
</tr>
<tr>
<td>$\mu_{sb}$</td>
<td>$5.53 \times 10^{-9}$ km$^2$/s$^2$</td>
</tr>
</tbody>
</table>

Table 2: Stock coefficients for a triaxial ellipsoid with $a = 11$, $b = 10$, $c = 9$ (values taken from Table 1).

<table>
<thead>
<tr>
<th>$C_{nm}$</th>
<th>Formula</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$C_{20}$</td>
<td>$\frac{1}{15} (a^2 - b^2)$</td>
<td>-0.059396640516508</td>
</tr>
<tr>
<td>$C_{22}$</td>
<td>$\frac{6}{25}(a^2 - b^2)$</td>
<td>+0.010507588566497</td>
</tr>
<tr>
<td>$C_{40}$</td>
<td>$\frac{15}{7}(C_{20}^2 + 2C_{22})$</td>
<td>+0.008038790549854</td>
</tr>
<tr>
<td>$C_{42}$</td>
<td>$\frac{1}{3} C_{20} C_{22}$</td>
<td>-0.000448469606523</td>
</tr>
<tr>
<td>$C_{44}$</td>
<td>$\frac{1}{2} C_{22}^2$</td>
<td>+0.000069835839151</td>
</tr>
</tbody>
</table>

3. Solar Sail Vicinity Operations

In this first section, we focus on the dynamics of a solar sail that is orbiting around the asteroid, where the main perturbing forces are the SRP and the Sun and the asteroid gravity field. As we are far from the asteroid, we can neglect its shape and approximate it as a point mass. Our goal is to describe the possible natural orbits in the system that can be interesting for monitoring the asteroid, with a fixed sail orientations.

3.1. Far Field Dynamics: Hill 3-Body Problem

The Augmented Hill 3-Body problem (AH3BP) models the motion of an infinitesimal particle (sail-craft) that interacts with a small mass (asteroid) and is perturbed by a distant large body (the Sun), and takes into account the effect of solar radiation pressure (SRP). As the Sun-sail distance is large compared to the asteroid-sail distance, the effect of the Sun is included as a pressure (SRP). As the Sun-sail distance is large compared to the Sun, and takes into account the effect of solar radiation pressure (SRP), this is the acceleration given by the solar sail.

\[
\frac{\partial \vec{r}}{\partial t} = \frac{P \vec{n}}{m} (r_{sb})^2, \quad (2)
\]

where, $P = P_0/R_0$ is the SRP magnitude at a distance $R$ from the Sun ($P_0 = 4.563 \mu N/m^2$, the SRP magnitude at $R_0 = 1$ AU), $A$ is the area of the solar sail, $r_{sb}$ is the distance of SRP at $(1,0,0)$ in the rotating reference frame and $\vec{n}$ is the normal direction to the surface of the sail.

The sail’s orientation is parameterised through two angles $\alpha$ and $\delta$, which represent the horizontal and vertical displacement of $\vec{n}$ with respect to $r_{sb}$. Hence $\vec{n} = (\cos \alpha \cos \delta, \sin \alpha \cos \delta, \sin \delta)$. Notice that $a_{\text{sail}}$ cannot point towards the Sun, hence $\alpha, \delta \in [-\pi/2, \pi/2]$.

The sail’s efficiency is measured in terms of the sail lightness number $\beta = 2PA/m$. As we use in Eq.(1) normalised units for distance and time these parameter $\beta$ has to be re-scaled, having $\bar{\beta} = \beta(\mu_{sb}/\omega_0)^3 - 1/3$ as the normalised sail lightness number. Following the definition of Broschart et al. and Giancotti et al. we have $\hat{\beta} = K_1(A/m)\mu_{sb}^{3/2}$ where $K_1 = 7.6874$ when $A$ is given in $m^2$ and $m$ in kg. Through this paper we consider that the solar sail has a total area of $50 \times 50 m^2$ and a total mass of $1300$ kg that orbits around 2001 DY103. Taking the asteroid’s parameters from Table 1 we have that $\bar{\beta} = 179.44$.

3.2. The Jacobi constant

An important property of the AH3BP is that the system is Hamiltonian and admits an integral of motion, also known as the Jacobi constant.

\[
J_c = \dot{X}^2 + \dot{Y}^2 + \dot{Z}^2 - 2\Omega(X, Y, Z) + a_X X + a_Y Y + a_Z Z. \quad (3)
\]

For a given initial condition, $J_c$ is preserved through time. This constant is related to the energy of the system and it provides an insight of different types of motion, as well as the regions where the motion is possible. Notice that $J_c = ||\vec{v}||^2 - 2\Omega + (a_{\text{sail}}, \vec{r})$, hence the regions of possible motion are delimited by the zero velocity surfaces $\{X, Y, Z\} = \{X, Y, Z\} \subset \mathbb{R}^3 | J_c + 2\Omega - (a_{\text{sail}}, \vec{r}) = 0\}$. In Fig. 1 we show these curves for a solar sail that is perpendicular to the Sun-sail line (i.e. $\alpha = \delta = 0$) and $\bar{\beta} = 179.44$. Notice how, for low energy levels, $J_c \leq -60$, the motion around the asteroid and inside the asteroid are separated, but as this one increases, $J_c \geq -50$, a gap is opened at $L_2$ connecting the inner and outer regions.

3.3. Equilibrium Points

We recall that when we neglect the effect of the solar sail ($\beta = 0$) the system has two equilibrium points $L_1$ and $L_2$ symmetrically located around the origin, whose coordinates are $(\pm 3^{1/3}, 0, 0)$. When we include the SRP ($\beta \neq 0$) and the sail
3.4. Periodic Orbits

Previous works\(^2\) show that around the displaced equilibria (\(SL_2\)) we can find periodic and quasi-periodic orbits that spend more than half of the time on the day side of the asteroid, which would be useful for mission operations. Here we want to briefly describe some of those orbits for \(\tilde{\beta} = 179.44\). We will also see how changes of the sail orientation affect this orbits.

When we consider the sail perpendicular to the Sun-sail line, we are essentially changing the gravitational pull from the Sun an the sail-craft. The non-linear dynamics close to the displaced equilibrium point is very similar to the no-sail case. The motion on the \(Z = 0\) plane is decoupled from the rest, and one of the families that emanates from the equilibrium point is completely contained in the plane. We call them planar Lyapunov orbits. The other family oscillates above and bellow the \(Z = 0\) plane, and we call them vertical Lyapunov orbits. As it happens in the classical Hill problem, when we move along the planar family, at some point we have a 1:1 resonance between the orbit and its normal frequencies and the Halo family of periodic orbits appears. In Fig. 2, we show the \(xyz\) projection of the orbits in these three families. Notice how some of the orbits in both the planar and vertical Lyapunov families spend most of their orbital period on the day side of the asteroid. These kind of orbits can be used to observe the asteroid. On the other hand, although the Halo get very close to the asteroid they remain on the dark side of the asteroid. If we look at the linear stability of these orbits we can see that most of them are linearly unstable except the final orbits on the Halo family that are linearly stable. The stable ones are known in the literature as Terminator orbits and are plotted in blue in Fig. 2. Broschart et al.\(^2\) saw that along the Terminator orbits there are several n:m resonances that give rise to periodic orbits that spend most of their time on the day side of the asteroid.

As it happens with the equilibrium points, we can artificially displace the periodic orbits by changing the sail orientation. Changes in \(\alpha\) induce an extra force on the \(xy\) plane and the orbits are displaced to one side or the orbits of the Sun-asteroid line. Changes in \(\delta\) induce an extra force on the \(z\) direction and displace the orbits above and below the asteroid’s orbital plane. Both changes will break, in some cases, the symmetry of the orbits.

To show this phenomena we have taken one orbit from each family that is close to the asteroid, and used a continuation method to compute their displaced equivalents. We have varied \(\alpha \in [-15^o, 15^o]\), \(\delta = 0\) and \(\alpha = 0, \delta \in [-15^o, 15^o]\) keeping the same orbital period for all the displaced orbits. Fig. 3 shows the results for the three different families. Notice how the displaced Terminator orbits spend part of their orbital period on the day side.

Being able to displace the periodic orbits with a solar sail is interesting for two main factors. The first one is that this will increases the visibility properties of the orbits and their potential use for observational missions. Moreover, we can use an active control law to drift along these families of displaced orbits following the ideas presented in Farrés and Jorba.\(^5\,^6\)

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\(^2\) Broschart et al.  
\(^5\,^6\) Farrés and Jorba.
Fig. 3: Periodic orbits for \( \alpha \in [-15^\circ, 15^\circ], \delta = 0 \) (left) and \( \alpha = 0, \delta \in [-15^\circ, 15^\circ] \) (right) of the planar Lyap. (top) vertical Lyap. (middle) and Halo (bottom) families.

4. Landing Operations

The focus of this section is to investigate the dynamics of the lander after its deployment from the solar power sail. In particular, we want to explore how to confine the lander dynamics in the case of bouncing motion, to prevent the falling off of the lander from the asteroid surface. In this case, the lander is in the proximity of the asteroid (closed field dynamics) where the SRP can be neglected. The lander dynamics is influenced mainly by the irregular shape of the asteroid here approximated by a triaxial ellipsoid. JAXA's solar power sail will be first placed hovering at 40 km from the asteroid surface (Home Position (HP) 1), pointing at the Earth to enhance communication. The solar power sail will then descend to 1 km from the surface (HP 2) to deploy the lander as shown in Fig. 4. We have explored the possibility of a free-falling trajectory for the sail from 40 km to 1 km, considering the sail to be Sun-pointing. Notice that when the Earth-Asteroid-Sun angle is small the HP from 40 km to 1 km, considering the sail to be Sun-pointing. In this case, the sail would reach 1 km distance from the surface with a residual velocity of 8.42 m/s. However, a controlled descend of the sail is likely to be adopted for safety reasons.

4.1. Near Field Dynamics: Perturbed 2-Body Problem

The equations of motion in the close vicinity of the asteroid are given in the two-body problem with the gravity harmonics expressed in the rotating (ACAF) frame, where the spin axis is aligned with the z-axis:

\[
\begin{align*}
\ddot{x} - 2\omega_{sb}\dot{y} &= \frac{\mu_{sb}}{r^3} x + \omega_{sb} \omega_{sb} x + \frac{\partial B}{\partial x}, \\
\ddot{y} + 2\omega_{sb}\dot{x} &= \frac{\mu_{sb}}{r^3} y + \omega_{sb} \omega_{sb} y + \frac{\partial B}{\partial y}, \\
\ddot{z} &= \frac{\mu_{sb}}{r^3} z + \frac{\partial B}{\partial z},
\end{align*}
\]

where \( \omega_{sb} \) is the angular velocity of the spin axis (2\( \pi \)/\( \tau \) in Table 1) and \( B \) is the potential associated to the asteroid harmonics defined as: \(^{17}\)

\[
B(r, \phi, \lambda) = -\frac{\mu_{sb}}{r} \left\{ \sum_{n=1}^{\infty} \left( \frac{r_{sb}}{r} \right)^n \sum_{m=1}^{n} \left( C_{nm} \cos(m\lambda) + S_{nm} \sin(m\lambda) \right) \right\},
\]

where \( n \) and \( m \) are respectively the order and the degree of the harmonics taken into account; \( r, \phi \) (latitude) and \( \lambda \) (longitude) are the coordinates of the satellite in spherical coordinates with respect to the ACAF frame; \( P_{nm} \) are the Legendre polynomials; while \( C_{nm} \) and \( S_{nm} \) are the Stockes coefficients. Note that \( B \) is expressed in spherical coordinates \( (r, \phi, \lambda) \), while the accelerations in Eq.(4) are expressed in Cartesian ACAF coordinates \( (x, y, z) \). Thus, a transformation from spherical to Cartesian coordinates is required. \(^{17}\)

The energy of the spacecraft is given by:

\[
E = \frac{1}{2} (\dot{x}^2 + \dot{y}^2 + \dot{z}^2) - \frac{1}{2} \omega_{sb}^2 (x^2 + y^2) - \frac{\mu_{sb}}{r} - B. \quad (6)
\]

Fig. 5 shows the shape of the potential energy (when the velocity in Eq.(6) is equal to zero) as a function of \( r \) and \( \lambda \) for \( \phi = 0^\circ \) (i.e. at the equator).

Fig. 4: Landing phase mission architecture: HP1 at 40 km (sail and lander) and HP2 at 1 km (lander deployment).

Notice that Eq.(4) is an autonomous system that admits four equilibrium points (1:1 resonances between the asteroids spin rate and the equatorial orbit around the asteroid). \(^{12}\) For a triaxial ellipsoid, the four equilibrium points lie on the equator \( (\phi = 0^\circ) \) and are located at \( \lambda \) equal to \( 0^\circ, 90^\circ, 180^\circ \) and

Fig. 5: Potential Energy as function of \( r \) and \( \lambda \) for \( \phi = 0^\circ \).
270°\(^\circ\).\(^{15}\) The equilibrium point can be found by imposing the velocities and accelerations equal to zero in Eq.(4). The coordinates of the equilibrium points are shown in Fig. 6 where \(P_{1,3} = (±26.61, 0, 0)\) km with \(E = -3.184 \times 10^{-5}\) and \(P_{2,4} = (±26.466, 0, 0)\) km with \(E = -3.172 \times 10^{-5}\).

As for the Hill problem, the Zero Velocity Curves (ZVC) of the perturbed two-body problem (also known as Roche Lobe\(^{16}\)) give qualitative information on the lander motion. We recall that, if the energy of the lander is less than the energy of \(P_{1,3}\), then the ZVC separate the inner from the outer motion. In this case the motion around the asteroid will be confined if the lander is inside this region. On the other hand, when the energy of the lander increases, a gap close to \(P_{1,3}\) is opened, connecting the inner and outer regions (green curves in Fig. 6). In this case the trajectory of the lander can escape. Note that the energy correspondent to an escape trajectory (i.e. green curves in Fig. 6) is associated to a velocity of the lander that is usually lower then the asteroid escape velocity.\(^{15}\) In Fig. 6 we show the ZVCs for different energy values: in black we have the energy value for \(P_{1,3}\); the red and blue lines correspond to energy values lower than \(P_{1,3}\) (i.e. \(-3.438 \times 10^{-5}\) and \(-4.9 \times 10^{-5}\) respectively); and the green curve represent the case where the energy is higher than the one at \(P_{1,3}\).

Finally, when the ZVCs are closed, we denote as altitude of the ZVC the point on the ZVC that intersects the negative \(z\)-axis and is on the right-hand side of \(P_{1}\). This definition is used when exploring the possible release velocity of the lander as a function of the maximum altitude within the lander can bounce. For example, \(P_{1}\) is the minimum altitude to close the ZVC and its ZVC altitude corresponds to -26.61 km (black point on Fig. 6), while the red point corresponds to a ZVC altitude of -20 km and the blue point correspond to a ZVC altitude of -12.5 km.

Fig. 6: Zero Velocity Curves for a triaxial ellipsoid.

4.2. Lander Trajectory

The altitude of deployment is fixed at 1 km, and given the fact that the maximum radii for the asteroid is of 11 km, we will consider that the initial condition for the deployment is at \((-12, 0, 0)\) km. Then, the deployment and touchdown velocities of the lander can be determined as a function of the altitude of the ZVC (or the energy) as defined in the previous section.

Fig. 7(a) shows the deployment velocity for altitude of the ZVC that increases from -12 km (deployment point) to -26.61 km \((P_{3})\). When the altitude of the ZVCs is at -12 km, the deployment velocity is 0 m/s, which represents potentially the best case scenario. However, a higher velocity can be required to compensate uncertainties in the solar power sail position or for landing time constrains. Thus, Fig. 7(a) tells the maximum potential bouncing altitude of the lander as a function of the deployment velocity when the velocity is given perpendicular to the asteroid surface (nominal case). Fig. 7(b) shows the correspondent touchdown velocity when the deployment velocity is set. The worst case scenario of elastic bouncing (no dissipation) is here considered where the touchdown velocity corresponds to the take-off velocity after the first touchdown. Along both curves in Fig. 7 we marked the case highlighted in Fig. 6, where the blue star correspond to a ZVC altitude of -12.5 km, the red star to -20 km, while the black star is the minimum ZVC altitude that guarantees bounded motion corresponding to \(P_{3}\).

Note that the velocities in Fig. 7 are express in the Asteroid-Centred Inertial (ACI) frame. For the case of small Earth-Asteroid-Sun angle, we can directly correlate the ACI frame with the Hill coordinates. Thus, the \(\Delta V\) given is the one required by the solar power sail. We recall that the rotation between the Asteroid-Centred Asteroid-Fixed (ACAF) frame and the Asteroid-Centred Inertial (ACI) frame is given by a rotation along the \(z\)-axis, \(R_{z}(\phi)\), with \(\phi = \omega \cdot t + \phi_{0}\) so that \(\mathbf{r}_{ACAF} = R_{z}(\phi) \cdot \mathbf{r}_{ACI}\).\(^{17}\)

The lander deployment velocity is here explored for different initial conditions including when the ZVC can or cannot bound the motion of the spacecraft around the asteroid as shown in Fig. 8. In Table 3 we summarise some of the cases that we have studied where in the first column we show the initial velocities \((t_{i})\) at which the lander is realised and in the second column we show the final velocities at touchdown \((t_{f})\). In Fig. 8 we show the trajectory that the lander follows for the different initial velocities. In Fig. 8, two different trajectories are shown: the nominal trajectory when the velocity of the lander is along the \(x\)-axis (light blue line) and one general case where the velocity components of the lander have \(x\) and \(y\) components in a 45° direction (black line) and an elastic bouncing is also considered. This case was selected to demonstrate the importance of having bounded motion. Each row of figures in Fig. 8 corresponds to a different initial deployment velocity (first column
Fig. 8: Deployment velocity of the lander at 1 km from the surface: light blue trajectory when the velocity is along the $x$-axis and black trajectory when the velocity has both $x$ and $y$ components.

(a) Deployment velocity of the lander 2.14 m/s. Left zoom relevant region.

(b) Deployment velocity of the lander 5.82 m/s. Left zoom relevant region.

(c) Deployment velocity of the lander 6.237 m/s. Left zoom relevant region.

(d) Deployment velocity of the lander 6.24 m/s. Left zoom relevant region.

(e) Deployment velocity of the lander 8.88 m/s. Left zoom relevant region.

(f) Deployment velocity of the lander 9.94 m/s. Left zoom relevant region.

Table 3: Deployment and Touchdown velocities for the cases in Fig. 8

<table>
<thead>
<tr>
<th>$V(t_0)$ [m/s]</th>
<th>$V(t_f)$ [m/s]</th>
</tr>
</thead>
<tbody>
<tr>
<td>2.14</td>
<td>6.65</td>
</tr>
<tr>
<td>5.82</td>
<td>8.63</td>
</tr>
<tr>
<td>6.237</td>
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<tr>
<td>6.24</td>
<td>8.87</td>
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<tr>
<td>8.88</td>
<td>10.89</td>
</tr>
<tr>
<td>9.94</td>
<td>11.77</td>
</tr>
</tbody>
</table>

Note that the ZVCs are an important dynamical tool when choosing the landing site. In case the when the ZVCs touch the surface, it is possible to constrain the bouncing motion of the lander on one side of the asteroid for example the day side. Fig. 9 shows the case in which the lander is drooped from the positive $y$-axis at 1 km from the surface with an initial velocity of 2.75 m/s. In this case, the lander motion is constrained in the positive coordinates of the $y$-axis. Hence, for a uniformly rotating asteroid, the ZVCs are an interesting tool for the selection of the landing point. This type of study can be generalised to more complex shape through the definition of the potential for polyhedron. The ZVCs were studied for Phobos where the the minimum energy for bounded motion of Phobos shows that surface is prone to escape dynamics. Scheeres studied the ZVC for Eros and Itokawa to show the velocity distribution on the their irregular surface. These considerations should be kept in mind when selecting the target asteroid for landing.

We want to mention that, the elastic bouncing was computed by introducing a local horizon frame. The ACAF frame with $x$ and $y$ coordinates, while the local horizon frame is given by...
\( x' \) and \( y' \) (see Fig. 10). Where \( V_1 \) is the velocity at touchdown while \( V_2 \) is the velocity at lift off. We call \( \alpha \) the angle between the touchdown position \((d)\) and \( V_1 \), and \( \gamma \) the angle between the touchdown position \((d)\) and the local horizon \( x \) coordinate. So:

\[
\alpha = \cos^{-1} \left( \frac{V_1 \cdot d}{||V_1|| \cdot ||d||} \right), \quad V_2 = ||V_1|| \left[ \begin{array}{c} \cos(\alpha) \\ \sin(\alpha) \end{array} \right]. \quad (7)
\]

\[
\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} \cos(\gamma) & \sin(\gamma) \\ -\sin(\gamma) & \cos(\gamma) \end{bmatrix} \begin{bmatrix} x' \\ y' \end{bmatrix}. \quad (8)
\]

Fig. 10: ACAF (left) and local horizon (right) reference frame at the touchdown position.

5. Conclusions

This paper focuses on the JAXA’s solar power sail mission to Trojan asteroids with particular interest on the asteroid vicinity operations. The dynamics of the solar power sail and of the lander are here investigated following the assumption that the solar power sail is mainly influenced by the SRP effect while the lander dynamics are mainly perturbed by the irregular shape of the asteroid. Thus, it is possible to distinguish between the asteroid far and close field dynamics. Under this assumption, the sailcraft follows the far field dynamics approximated in the Hill problem while the lander is operating in the close field dynamics where its dynamics are approximated with a perturbed two body problem. For the case of the sail, it is investigated orbit solutions as an alternative of JAXA’s hovering baseline where the orbits are obtained as a function of the attitude of the sail. This methodology adds mission flexibility depending on the mission constraints. The lander operation has been studied looking at the ZVCs as an important tool for landing decision making and for confining the lander trajectory by preventing the fall-off of the lander from the asteroid surface after bouncing.

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