Radiation Pressure Force Model for an Ideal Laser-Enhanced Solar Sail

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The concept of a laser-enhanced solar sail is introduced and the radiation pressure force model for an ideal laser-enhanced solar sail is derived. A laser-enhanced solar sail is a “traditional” solar sail that is, however, not solely propelled by solar radiation, but additionally by a laser beam that illuminates the sail. The additional laser radiation pressure increases the sail’s propulsive force and can give, depending on the location of the laser source, more control authority over the direction of the solar sail’s propulsive force vector. This way, laser-enhanced solar sails may augment already existing solar sail mission concepts and make novel mission concepts feasible.

Key Words: Laser-Enhanced Solar Sail, Force Model

Nomenclature

- $A$: sail area
- $c$: speed of light in vacuum
- $\hat{d}$: unit vector into the desired thrust direction
- $F$: thrust force
- $\hat{f}$: unit vector in laser-sail direction
- $L$: solar radiation flux
- $\hat{n}$: sail normal (unit) vector, perpendicular to the sail surface in the direction away from the Sun
- $r$: solar sail distance
- $\hat{r}$: unit vector in Sun-sail direction
- $S$: solar radiation flux
- $\hat{s}$: transversal unit vector, $\hat{t}\perp\hat{r}$
- $T$: sail temperature
- $\varepsilon$: sail emissivity
- $\kappa$: combined thermo-optical parameter
- $\rho$: sail reflectivity
- $\sigma$: Stefan-Boltzmann constant

Subscripts

- $0$: reference (1 AU)
- $b$: sail backside
- $f$: sail front side
- $L$: laser
- $\text{max}$: maximum value
- $S$: solar

1. Introduction

For a perfectly reflecting solar sail, both the magnitude and the direction of the solar radiation pressure (SRP) thrust force are completely determined (and thus also constrained) by the solar distance $r$ and the orientation of the sail with respect to the Sun. The SRP thrust force decreases with $1/r^2$ and, even close to the Sun, the thrust component into the desired thrust direction is typically smaller than the component perpendicular to the desired direction. This circumstance limits the maneuverability of “pure” solar sails and makes mission phases like an escape from a Low-Earth orbit or a rendezvous with an outer solar system body practically infeasible.

To enhance the maneuverability of “pure” solar sails and to extend their mission applicability, the concept of a laser-enhanced solar sail is proposed in this paper. This concept must not be confused with the very-high performance laser sails that were investigated for interstellar missions in the past (see e.g. the work by Forward and the references therein). A laser-enhanced solar sail is a “traditional” solar sail that is, however, not solely propelled by solar radiation, but additionally by a laser beam that illuminates the sail, as it is schematically sketched in Fig. 1.

![Fig. 1. Schematic sketch of a laser-enhanced solar sail. The thrust force $F_L$ results from the solar radiation, while the thrust force $F_s$ results from the laser radiation. Both forces are normal to the sail surface and, assuming that the laser illuminates the sail’s front side, directed away from the Sun ($\hat{F}\cdot\hat{n}\geq0$).](image_url)
solar system and its optimal orbit (or their optimal orbits) for a given mission is subject for future research and optimization.

This paper provides the force model for an ideal laser-enhanced solar sail and demonstrates that a laser can enhance the thrusting capability of a solar sail. The term “ideal” means that the solar sail is assumed to be perfectly reflecting and that the laser beam is assumed to have a constant areal power density over the whole sail area. Since a laser beam has a limited divergence, it can provide intense radiation at larger solar distances and thus increase the radiation pressure force into the desired direction. Therefore, laser-enhanced solar sails may make missions feasible that would otherwise have prohibitively long flight times, e.g. rendezvous missions with outer-solar-system bodies and missions into near-interstellar space. They may also enhance Earth-escape scenarios, which require very demanding attitude control profiles with rapid changes of the sail orientation.

2. Force Model for an Ideal Solar Sail

This section briefly revisits the force model for a perfectly reflecting solar sail, which is typically called an “ideal solar sail”. The SRP thrust force onto an ideal solar sail with area \( A \) is

\[
F_S(r) = 2p_S(r)A(\hat{r} \cdot \hat{n})^2 \hat{n}
\]

with the solar radiation pressure being

\[
p_S(r) = \frac{S(r)}{c} = \frac{S_0}{c} \left( \frac{r_0}{r} \right)^2
\]

Inserting Eq. (2) into Eq. (1) gives

\[
F_S(r) = 2 \frac{S_0}{c} \left( \frac{r_0}{r} \right)^2 A(\hat{r} \cdot \hat{n})^2 \hat{n}
\]

The reference distance \( r_0 \) is 1 AU, where the solar radiation flux is \( S_0 = 1368 \text{ W/m}^2 \), the so-called solar constant. The SRP thrust force is always perpendicular to the sail surface, along the so-called sail normal (unit) vector \( \hat{n} \). By definition, the sail normal vector points always into the hemisphere away from the Sun, i.e. \( \hat{r} \cdot \hat{n} \geq 0 \). The magnitude of the SRP thrust force depends not only on solar distance \( r \) but also on the orientation of the sail with respect to the Sun via \( \hat{r} \cdot \hat{n} \).

The location of all possible SRP thrust force vectors can be visualized by the so-called SRP-force “bubble”, on whose surface the tip of the SRP thrust force vector is constrained to lie, as shown exemplarily in Fig. 2.

![Fig. 2. Schematic sketch of a solar sail. The tip of SRP thrust force vector is constrained to lie on the surface of the dashed SRP-force "bubble".](image)

The SRP thrust force is maximal when the sail is perpendicular to the incoming solar radiation \( (\hat{r} \cdot \hat{n} = 1) \), but in this case, the direction of the thrust force is along the Sun-sail direction \( \hat{r} \) and has typically only a small component into the desired thrust direction \( \hat{d} \) (note that the thrust direction to change the orbital energy with a maximum rate is the velocity direction). The thrust into the desired direction can be maximized by maximizing \( F_S \cdot \hat{d} \), but the magnitude of the thrust force decreases proportionally to \( (\hat{r} \cdot \hat{n})^2 \).

For thermal reasons, most solar sail designs have a highly emissive backside. Note that such a sail can still be considered an ideal solar sail, as long as the front side is perfectly reflecting and the back side is not illuminated.

3. Model for an Ideal Laser-Enhanced Solar Sail

3.1. The Laser

The purpose of this paper is to provide a simple and generic model for a laser-enhanced solar sail that can be used for the investigation of the fundamental theoretical advantages that such a system offers. It is not within the scope of this paper to devise an engineering concept for the laser source. Therefore, it is assumed that the whole sail is homogeneously illuminated by laser beam, and that the laser beam has negligible divergence, so that the laser radiation flux \( L \) (in \( \text{W/m}^2 \)) is independent of the distance between the laser source and the sail. The laser should be switched off when it would decrease the thrust component into the desired direction or when it would lead to an excess of the maximal sail temperature (see Section 3.4).

3.2. Laser Radiation Pressure Force

According to the assumptions stated above for the laser source, the laser radiation pressure (LRP) thrust force exerted by a laser onto an ideal sail of area \( A \) is

\[
F_L = 2p_L A(\hat{\ell} \cdot \hat{n})^2 \hat{n}
\]

with

\[
p_L = \frac{L}{c}
\]

and \( \hat{\ell} \) being the unit vector in the laser-sail direction. The LRP thrust force is in the same direction as the SRP thrust force, along \( \hat{n} \), and its magnitude depends on the orientation of the sail with respect to the laser via \( \hat{\ell} \cdot \hat{n} \). Note that because of the assumption that the laser beam has negligible divergence, the LRP thrust force is independent from the laser-sail distance. Like for the solar radiation, the location of all possible LRP thrust force vectors is visualized by a LRP-force "bubble", on whose surface the tip of the LRP thrust force vector is constrained to lie, as shown exemplarily in Fig. 3.

![Fig. 3. Schematic sketch of a laser-enhanced solar sail with the relevant unit vectors and the force "bubbles" resulting from solar radiation and laser radiation.](image)
Note that the axis of symmetry is along $\hat{e}$ for the LRP-force "bubble", while it is along $\hat{r}$ for the SRP-force "bubble".

### 3.3. Total Radiation Pressure Force

The total radiation pressure force exerted by the combined solar and laser radiation onto an ideal sail of area $A$ is then obtained by adding Eqs. (3) and (4), as

$$F(r) = \frac{2A}{c} \left[ S_0 \left( \frac{\tau_0}{\tau} \right)^2 \left( \hat{r} \cdot \hat{n} \right)^2 + L(\hat{\ell} \cdot \hat{n})^2 \right] \hat{n}$$

or, denoting the ratio of the laser and the solar radiation flux $L/S(r)$ as $\lambda(r)$, as

$$F(r) = 2 \frac{S_0}{c} \left( \frac{\tau_0}{\tau} \right)^2 A \left[ \left( \hat{r} \cdot \hat{n} \right)^2 + \lambda(r)(\hat{\ell} \cdot \hat{n})^2 \right] \hat{n}$$

So, for a given sail, the total radiation pressure thrust force depends only on solar distance, laser radiation flux, and the orientation of the sail with respect to the Sun and the laser, respectively.

For the maximization of $F(r)$ into the desired thrust direction $\hat{d}$, the expression

$$\left[ \left( \hat{r} \cdot \hat{n} \right)^2 + \lambda(r)(\hat{\ell} \cdot \hat{n})^2 \right] (\hat{n} \cdot \hat{d})$$

must be maximized. There is, however, no "general solution" for the maximization of this expression, since, even for a constant orientation with respect to the Sun, i.e. a constant $\hat{r} \cdot \hat{n}$, $\hat{\ell}$ and $\lambda(r)$ vary with the changing position of the sail and probably also with the changing position of the laser source.

Figure 4 shows the thrust force vectors and the force "bubbles" for the simple case where the laser is located between the Sun and the sail. The laser radiation flux is taken as twice the solar radiation flux, i.e. $\lambda=2$. The orientation of the sail is to maximize the SRP thrust component into the transversal direction, given by the unit vector $\hat{e}$.

As Fig. 4 shows, the total thrust force is simply $1+\lambda=3$ times the "pure" SRP thrust force. The sail orientation that maximizes the transversal SRP force component also maximizes the total transversal force component.

Figure 5 shows the thrust force vectors and the force "bubbles" for the more general case where the laser is not located between the Sun and the sail. Again, $\lambda=2$. The orientation of the sail is still to maximize the SRP thrust component into the transversal direction.

As Fig. 5 shows, the orientation of the force "bubbles" is now different. The sail orientation that maximizes the transversal SRP thrust component does not maximize the total transversal thrust force component anymore. As a consequence, the optimal steering profile of the sail changes and the sail must fly with an orientation that is different to that of a "pure" solar sail, as shown in Fig. 6.

### 3.4. Maximal Laser Power

To prevent the thermal destruction of the sail film by overheating, the total radiation flux and thus the laser radiation flux on the sail must be limited. For the calculation of the sail temperature, the sail cannot be considered perfectly reflecting anymore, since such a sail would not absorb radiation power. Consequently, its temperature would always be 0K. Here, for simplicity, we assume for the thermo-optical parameters of the sail that

1. the thermo-optical coefficients for the solar spectrum and for the laser are the same
2. the transmissivity of the sail is zero. Thus, for a front-side reflectivity of $\rho=0.88$, the absorptivity is $1-\rho=0.12$
3. the front-side emissivity is $\varepsilon_f = 0.05$ and the backside emissivity is $\varepsilon_b = 0.55$
In thermal equilibrium, the absorbed radiation power is equal to the emitted radiation power:

\[
(1 - \rho) \left[ S_0 \left( \frac{T_0}{T} \right)^2 (\hat{F} \circ \hat{n}) + L(\hat{L} \circ \hat{n}) \right] A
= (\varepsilon_f + \varepsilon_b) \sigma T^4 A
\]

Using this equation, the temperature of a laser-enhanced solar sail can be calculated as

\[
T = \left[ \frac{1 - \rho}{\varepsilon_f + \varepsilon_b} S_0 \left( \frac{T_0}{T} \right)^2 (\hat{F} \circ \hat{n}) + L(\hat{L} \circ \hat{n}) \right]^{-\frac{1}{4}}
\]

where, in the second line, all temperature-relevant thermo-optical parameters are collected into a single parameter, \( \kappa = 0.20 \).

Using Eq. (10), the maximal laser radiation flux \( L_{\text{max}} \) can be calculated from the maximal sail temperature \( T_{\text{max}} \) as

\[
L_{\text{max}} = \frac{\sigma T_{\text{max}}^4 - S_0 \left( \frac{T_0}{T} \right)^2 (\hat{F} \circ \hat{n})}{\hat{L} \circ \hat{n}}
\]

For a given sail and solar distance, it depends only on the orientation of the sail with respect to the Sun and the laser, respectively.

For safety reasons, one should consider that the sail does not overheat even when its full area is exposed to both the solar and the laser radiation, i.e. \( \hat{F} \circ \hat{n} = \hat{L} \circ \hat{n} = 1 \), so that

\[
L_{\text{max}} = \frac{\sigma T_{\text{max}}^4 - S_0 \left( \frac{T_0}{T} \right)^2}{\kappa}
\]

Figure 7 shows a plot of the maximum laser radiation flux vs. solar distance for different sail temperature limits between 180°C and 240°C, calculated with Eq. (12).

Figure 8 shows the maximum ratio of the laser and solar radiation flux, \( \lambda_{\text{max}} \), for different sail temperature limits.

3.5. Potential System Challenges

In this paper, no engineering thought was given to the design of the laser source(s) but a very simple model was assumed for the laser beam (see Section 3.1). Keeping the laser beam almost parallel over large distances will be a challenge and probably require further optical components (e.g. a Fresnel lens). A small beam divergence could also be compensated by increasing the laser power, thereby reducing, however, the effectiveness of the whole system. Another major technical challenge will be the accurate pointing of the laser, to the position where the sail will be when the light arrives there.

4. Potential Applications

4.1. Outer Solar System Body Rendezvous

Despite the \( 1/r^2 \)-decrease of solar radiation pressure, solar sails allow also missions into the outer solar system.\(^4\) For those missions, the solar sail gains orbital energy in the inner solar system by so-called Solar Photonic Assist (SPA) maneuvers,\(^6,7\) leading to fast solar-system escape trajectories. Due to the low solar radiation pressure in the outer solar system, it is, however, not possible to break at a potential target body (e.g. a planet, a Trojan asteroid, or a Kuiper-Belt Object). Such missions may be possible for laser-enhanced solar sails, because their thrusting capability is not limited by solar distance. Lasers that are located in the outer solar system, e.g. at the L4 and L5 Lagrange points of Jupiter, could additionally expand the "thrust envelope" of solar sails.

4.2. Fast Solar System Escape

For fast solar-system escape trajectories, depending on the performance and temperature limit of the solar sail, the maximum solar system escape velocities are limited.\(^4\) A laser-enhanced solar sail would allow much higher solar system escape velocities and thus much shorter trip times into near-interstellar space. For such missions, the laser source may be located anywhere in the solar system.

4.3. Laser-Assisted Earth Escape

Leipold\(^3\) has shown that solar sail attitude control for Earth-escape missions is much more challenging than for interplanetary missions. Because, over one orbit, the SRP force "bubble" is always oriented into the same direction, there is always a part of the orbit, where the thrust cannot have a component into the desired direction, which is typically the velocity direction to gain orbital energy with a maximum rate. On this part of the orbit, the sail has to be oriented parallel to
the incoming sunlight. Figure 9 shows this behavior for different positions on an (assumed) circular orbit together with the SRP force "bubbles". A solar sail with a thermal coating on the back side that is intended to gain orbital energy must be turned by 180° in a very short time, which is extremely difficult due to the gossameriness and the high mass moment of inertia of the sail.

![Diagram of geocentric solar sail motion]

Fig. 9. Geocentric solar sail motion.

One or more laser satellites in the Earth-Moon Langrange points or in the right orbit may be used to enhance the Earth escape of a solar sail by making quick attitude maneuvers needless.

5. Summary and Conclusions

It was shown that, in principle, laser-enhanced solar sails have an advantage with respect to "traditional" solar sails, because the thrust is generally larger and can be directed into directions that are otherwise not possible. Given an optimized steering profile of the sail, this will reduce the flight time for a given sailcraft system mass or reduce the required system mass for a given flight time. Laser-enhanced solar sails may enhance already existing solar sail mission concepts and make novel mission concepts feasible.

The next step is to analyze exemplary mission scenarios without laying too much emphasis on the design and operations of the laser source. This includes the optimization of the location of the laser source(s). If the mission studies conclude that laser-enhanced solar sails would have advantages with respect to "traditional" solar sails, a detailed study of the laser source and the whole system architecture would be the second next step.

References